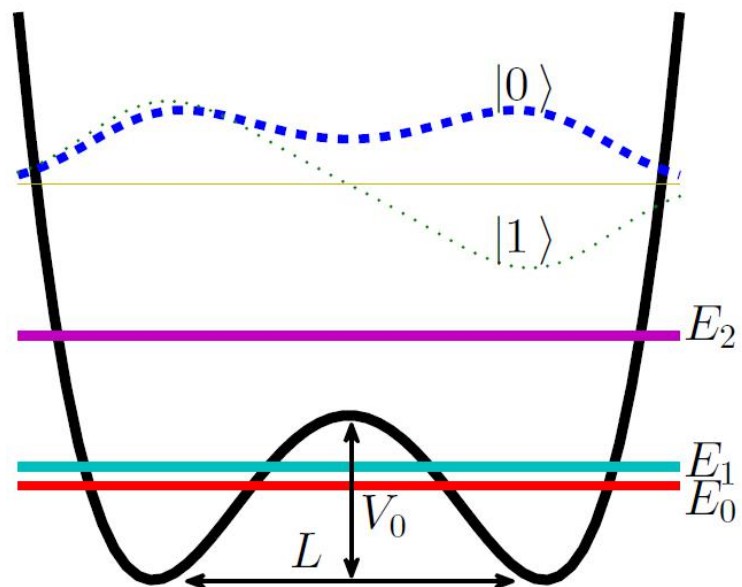
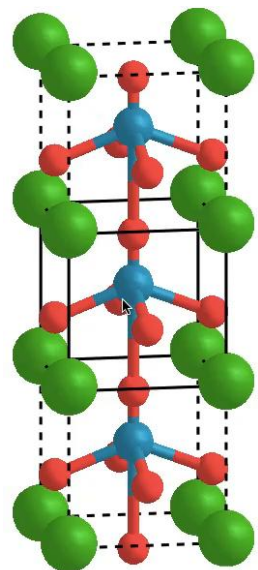
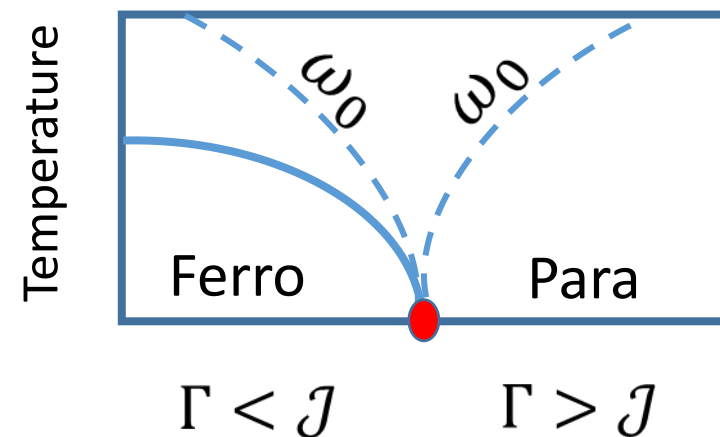


Ferroelectric modes



$$\Gamma = E_1 - E_0$$



Quantum Ising Model

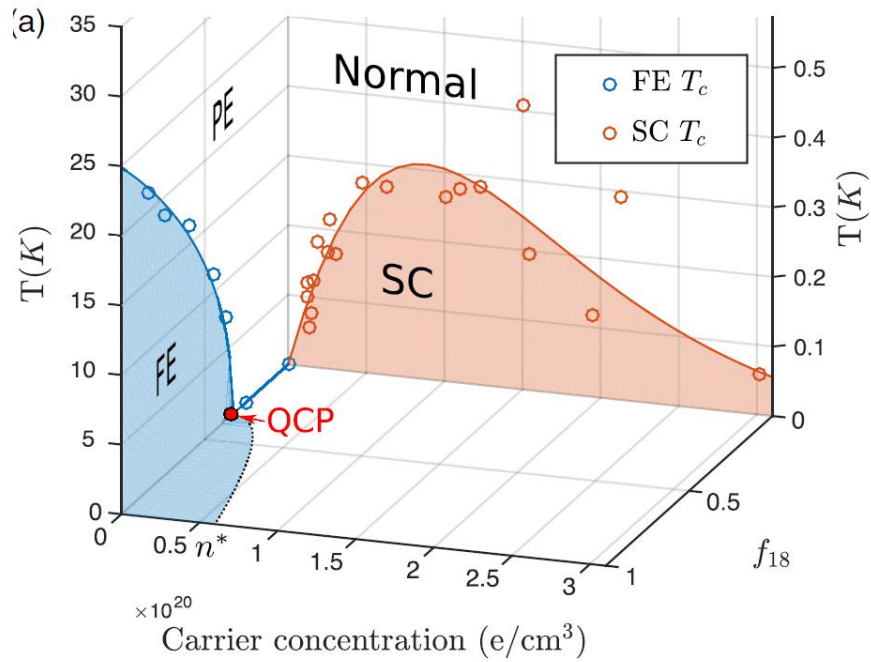
$$H_m = -\frac{\Gamma}{2} \sum_i \sigma_x(i) - \sum_{i,j} J_{i,j} \sigma_z(i) \sigma_z(j)$$

$$J_{\parallel} = \left[\begin{array}{|c|} \hline \text{Diagram 1} \\ \hline \end{array} \right] - \left[\begin{array}{|c|} \hline \text{Diagram 2} \\ \hline \end{array} \right]$$

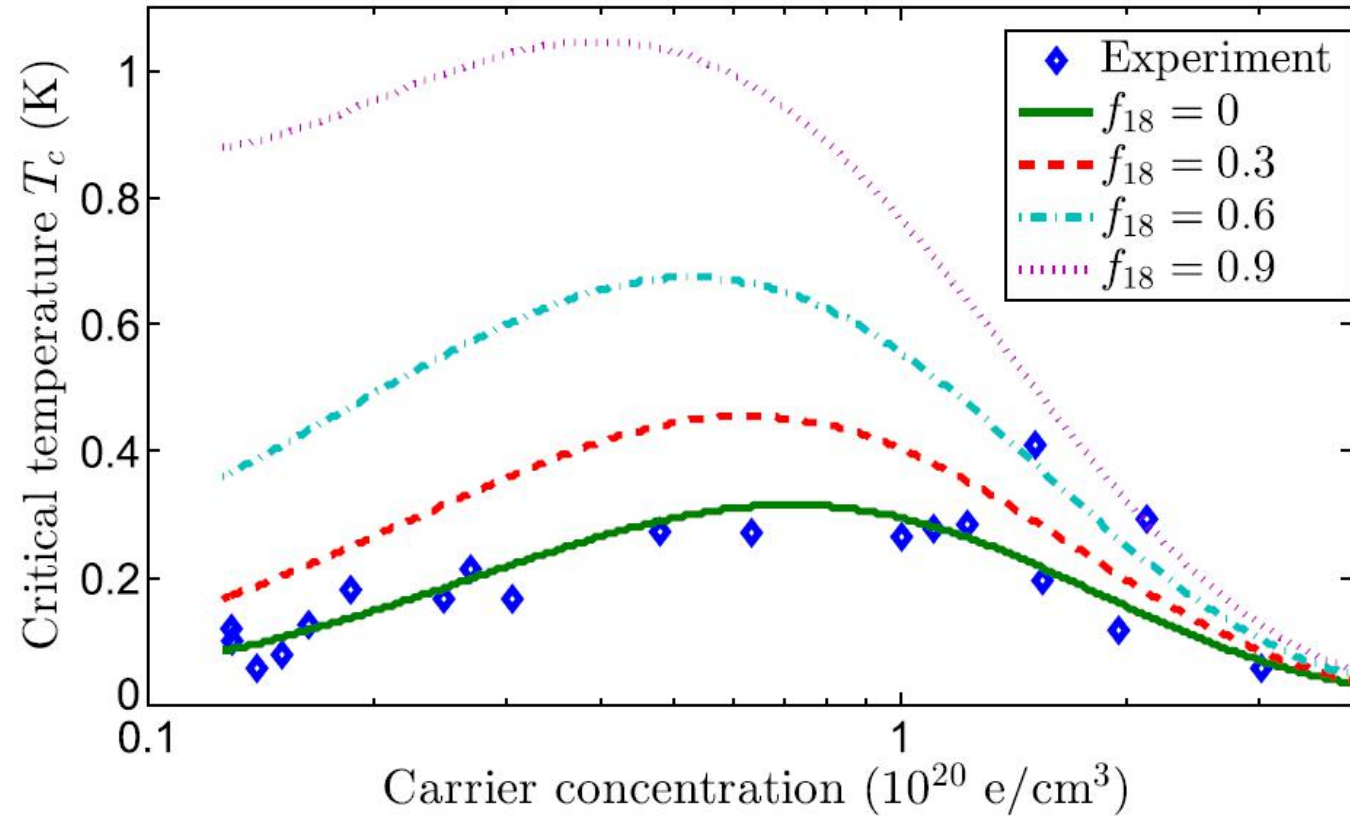
$$J_{\perp} = \left[\begin{array}{|c|} \hline \text{Diagram 3} \\ \hline \end{array} \right] - \left[\begin{array}{|c|} \hline \text{Diagram 4} \\ \hline \end{array} \right]$$

Edge, YK, Aschauer, Spaldin, Balatsky, PRL, 115, 247002 (2015)

YK, Zhu, Balatsky, PRB, 93, 184507 (2016)



Edge, YK, Aschauer, Spaldin, Balatsky, PRL, 115, 247002 (2015)



Qualitative predictions:

1. Heavier ions \rightarrow Higher T_c
2. 5% mass increase \rightarrow T_c doubles
3. Peak at lower doping

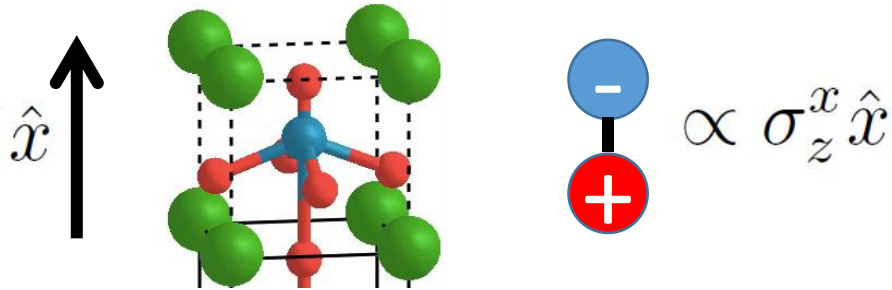
Confirmed:

Stuckey et. al. Scientific Reports 6, 37582 (2016)
 Tomioka et. al. Nat. Comm. 10, 738 (2019)

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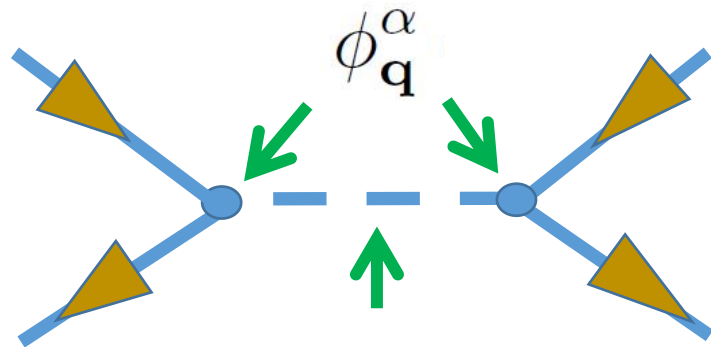
Coupling of the FE modes to mobile electrons

Electric dipole

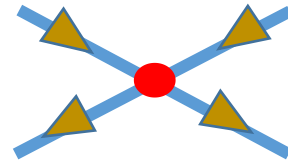
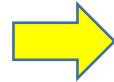


$$H_I = \sum_{\mathbf{k}, \mathbf{q}, \alpha} c_{\mathbf{k}}^\dagger c_{\mathbf{k}+\mathbf{q}} \sigma_z^\alpha(\mathbf{q}) \phi_{\mathbf{q}}^\alpha \quad \phi_{\mathbf{q}}^\alpha \propto \frac{q_\alpha}{q^2}$$

Coupling



Propagator



Effective retarded Interaction

$$V(\mathbf{q}, \omega) \propto \sum_{\alpha} \frac{|\phi_{\mathbf{q}}^\alpha|^2}{(\omega_{\mathbf{q}}^\alpha)^2 + \omega^2}$$

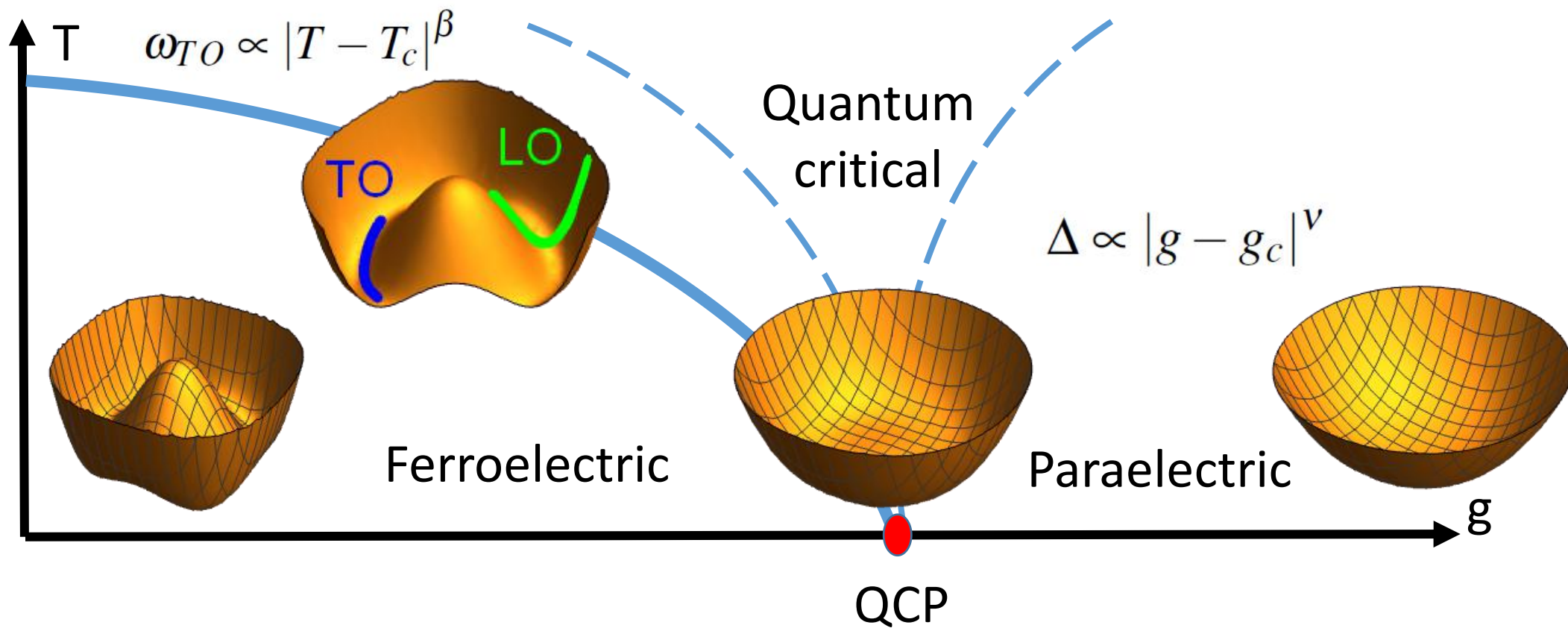
Self consistent equation

$$H_m^\alpha = -\Gamma \sum_i \sigma_x^\alpha(i) - \sum_{i,j} J_{i,j} \sigma_z^\alpha(i) \sigma_z^\alpha(j)$$

$$\Delta(\mathbf{k}, \omega) = -T \sum_{\mathbf{k}', \omega'} V(\mathbf{k} - \mathbf{k}', \omega - \omega') \frac{\Delta(\mathbf{k}', \omega')}{\Delta^2(\mathbf{k}', \omega') + \omega'^2 + \xi_{\mathbf{k}'}}^2$$

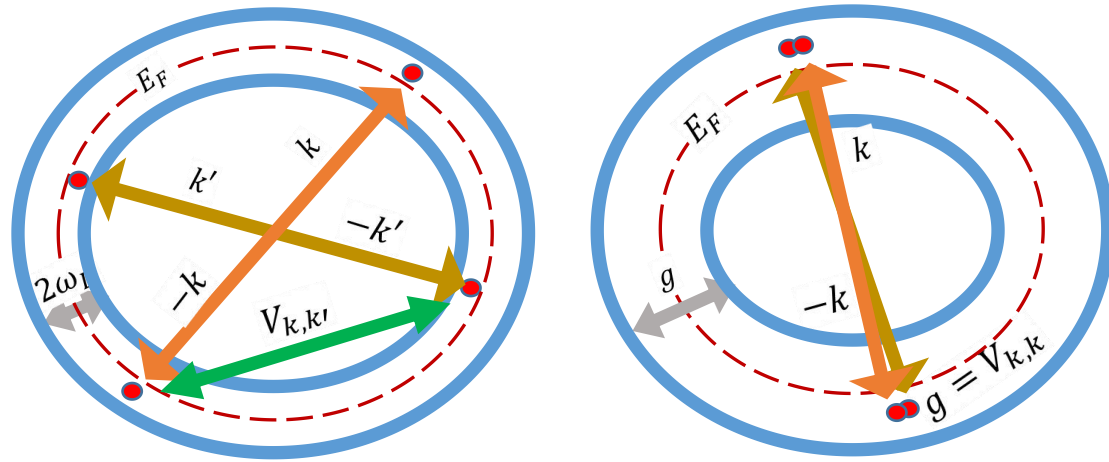
LST: $\frac{\omega_{LO}^2}{\omega_{TO}^2} = \frac{\epsilon_{st}}{\epsilon_{\infty}}$

YK, arXiv:2004.00029



Ultra low doping

YK, PRB 98 (22), 220505 (2018)



Adiabatic

Anti-adiabatic

Forward scattering: $V(\mathbf{q}, \omega) = g\delta(q)$

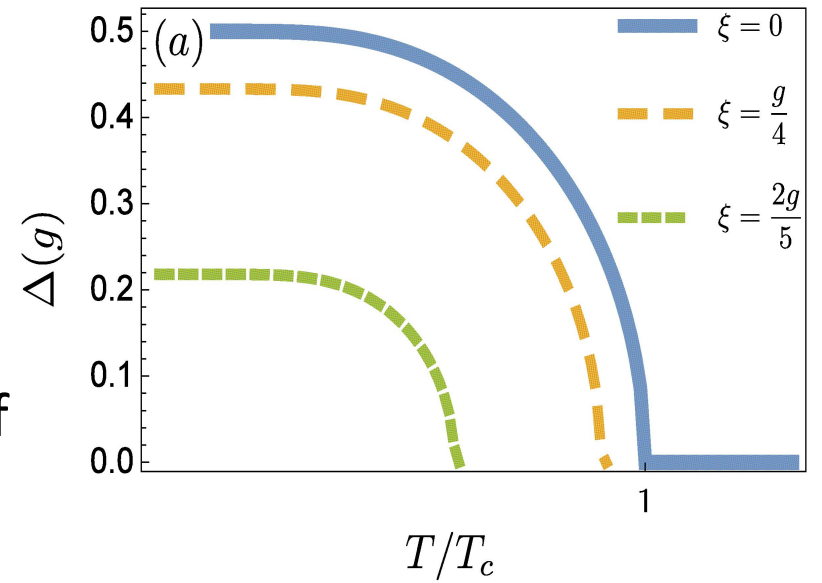
At $T=0$: $\Delta(\mathbf{k}) = \sqrt{g^2/4 - \xi_{\mathbf{k}}^2}$

At the Fermi surface: $T_c = g/4$

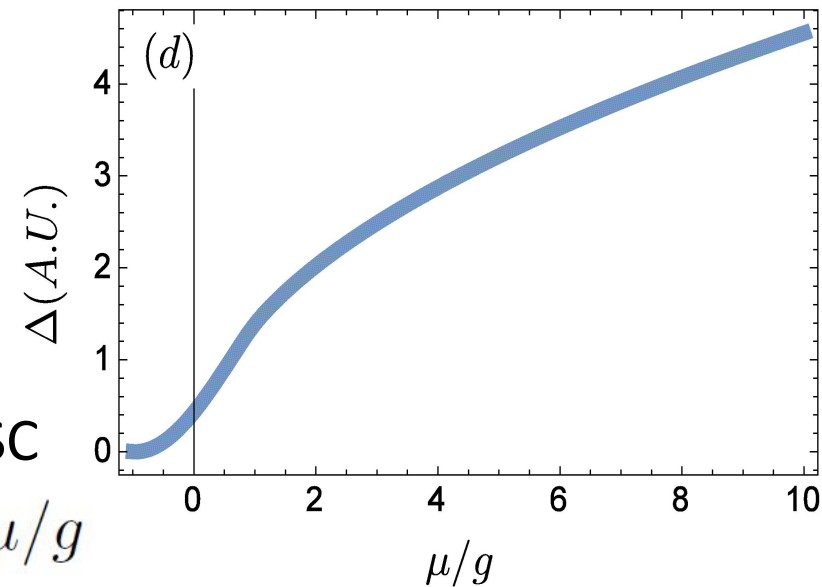
Predictions for low doping

1. T_c plateaus
2. Critical current and H_c vanish

Simplified Self Consistent Equation:



$$2\sqrt{\Delta^2(\mathbf{k}) + \xi_{\mathbf{k}}^2} = g \tanh\left(\frac{\sqrt{\Delta^2(\mathbf{k}) + \xi_{\mathbf{k}}^2}}{2T}\right)$$



Insulator SC

$$\Delta \propto 1 + \mu/g$$