Pairing in $SrTiO_3$: Aspects of the ferroelectric critical point

Jonathan Ruhman

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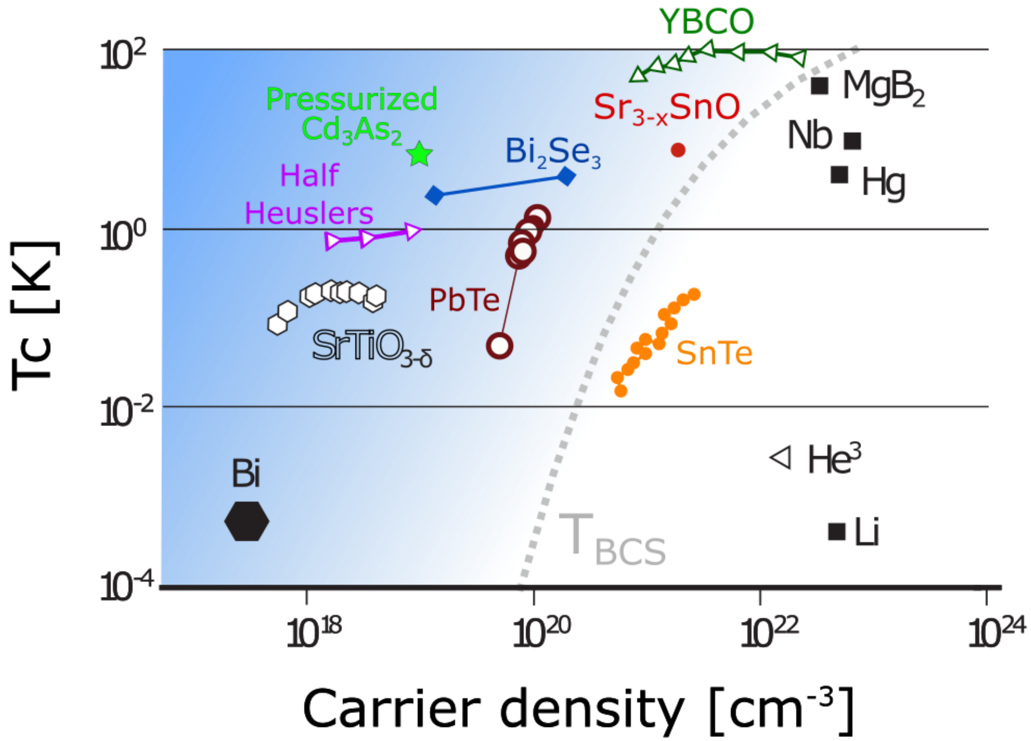
Superconductivity in doped materials

$$T_{\mathrm{BCS}} = \omega_D \exp[-1/N(0)V]$$



Kamran Behnia
Science 355, 26
Etienne Bustarret
Physica C, 514, 36

Lin et al PRL (2014)
Prakash et al Science (2018)
Butch PRB (2010)
Liu et al JACS (2015)
Matsushita et al PRB (2006)
He et. al. Nature (2016)

For bulk SC – only long-ranged interactions are important

Main candidates for pairing in STO

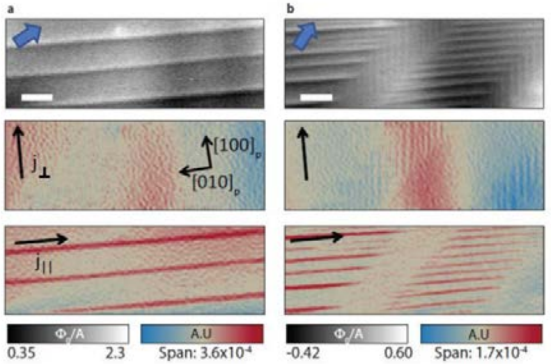
1. **Coulombic superconductivity:**

   The Gurevich-Larkin-Firsov (GLF) mechanism

2. **Lower dimensional / filamental superconductivity**

   (credit to Jeremy Levy)

$$N_{2D}(0) \sim \frac{m}{2\pi}$$



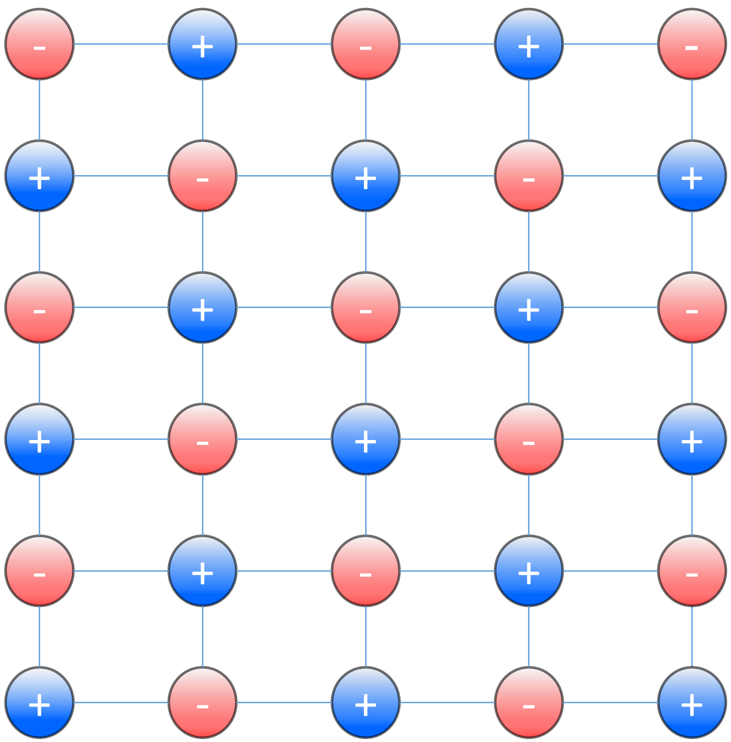
Kalisky et. al. Nat. Mat. (2014)

3. **Pairing from FE quantum fluctuations**

FE transition is a structural transition

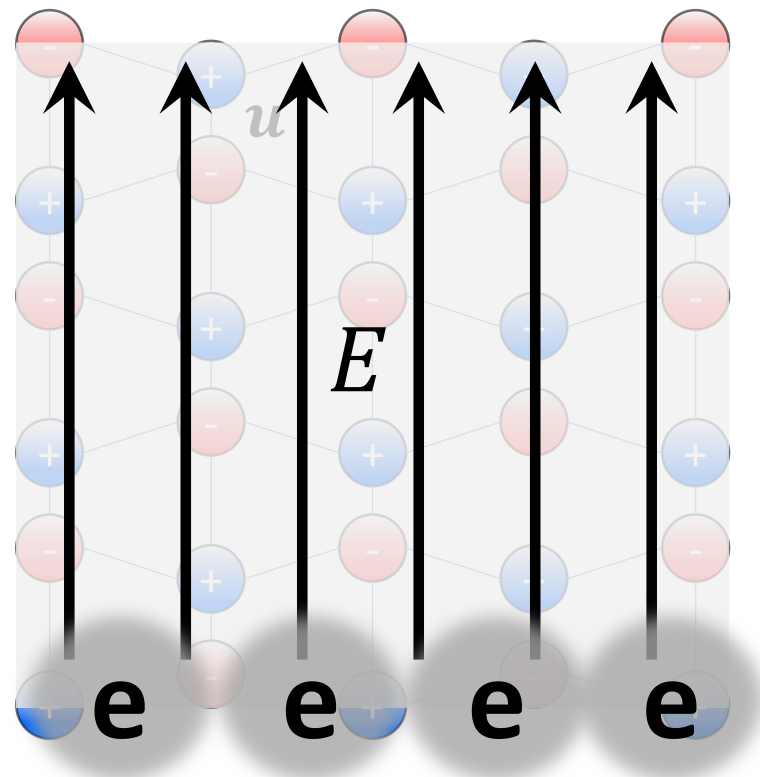
Paraelectric (disordered)

$$\langle u \rangle = 0$$



Ferroelectric (ordered)

$$\langle u \rangle \neq 0$$



Theory for FE QCP

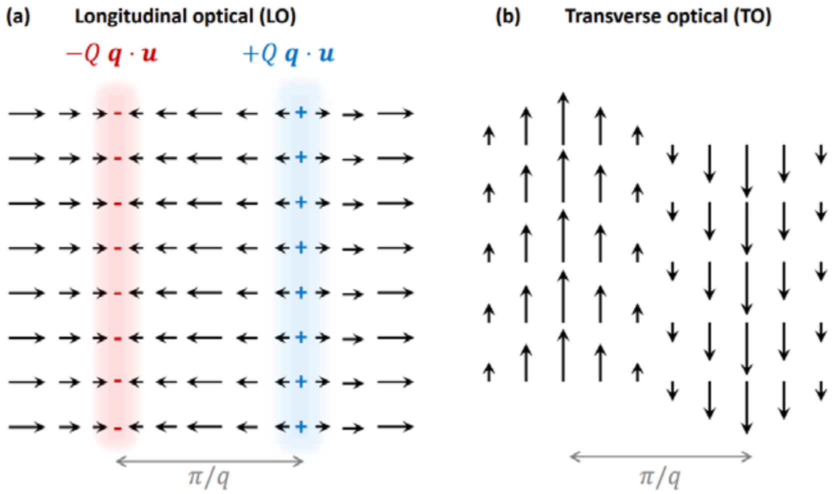
Khmelmitskii & Shneerson (1971)

$$L_u = u_l\left[\partial_t^2 - A_{jl}(\nabla)\right]u_j + \beta_1\left(u_x^4 + u_y^4 + u_z^4\right) + \beta_2\left(u_x^2 u_y^2 + u_x^2 u_z^2 + u_z^2 u_y^2\right)$$

$$A_{jl}(\boldsymbol{q}) = \omega_T^2 \delta_{jl} + \left(c_L^2 + \frac{\Omega_p^2}{q^2}\right) q_j q_l + c_T^2\left(q^2 \delta_{jl} - q_j q_l\right) + \alpha q_j^2 \delta_{jl}$$

Tuning parameter

Coulomb interaction



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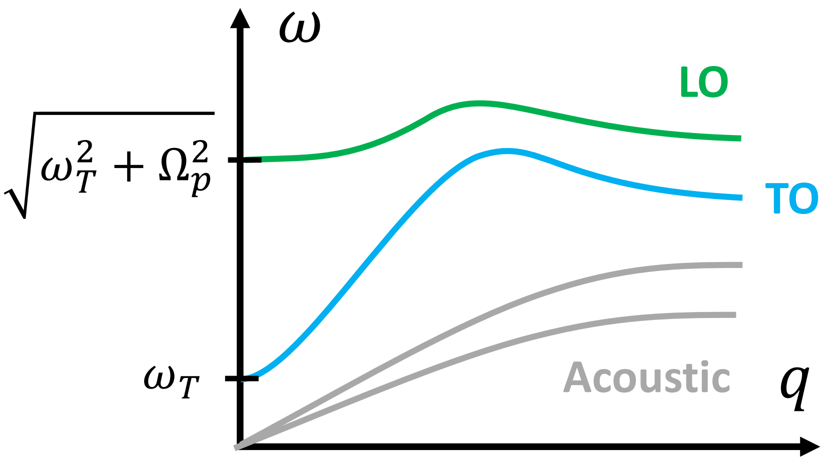
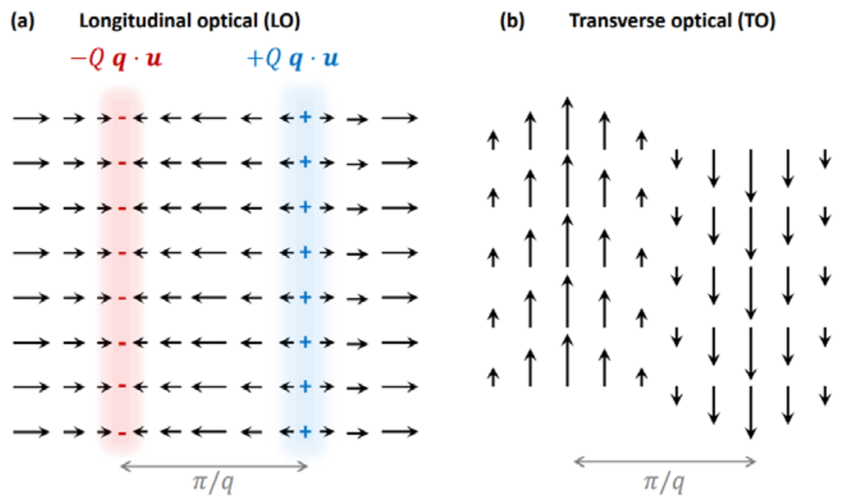
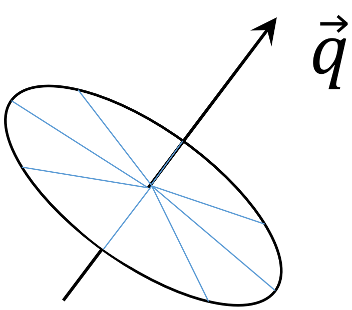
$$A_{jl}(\boldsymbol{q}) = \omega_T^2\delta_{jl} + \left(c_L^2 + \frac{\Omega_p^2}{q^2}\right)q_jq_l + c_T^2\left(q^2\delta_{jl} - q_jq_l\right) + \alpha q_j^2\delta_{jl}$$





The dipole-dipole interactions force the polarization vectors to become transverse in the $q \to 0$ limit (JR & Lee PRB 2019)

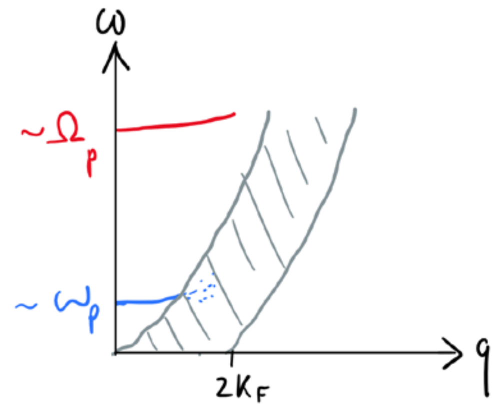


Theory for FE QCP – with electron doping

$$L_u = u_l\left[\partial_t^2 - A_{jl}(\boldsymbol{\nabla})\right]u_j + \beta_1\left(u_x^4 + u_y^4 + u_z^4\right) + \beta_2\left(u_x^2u_y^2 + u_x^2u_z^2 + u_z^2u_y^2\right)$$

$$A_{jl}(\boldsymbol{q}) = \omega_T^2\delta_{jl} + \left(c_L^2 + \frac{\Omega_p^2}{[1+\epsilon_e(\omega,\boldsymbol{q})]q^2}\right)q_jq_l + c_T^2\left(q^2\delta_{jl} - q_jq_l\right) + \alpha q_j^2\delta_{jl}$$

$$\epsilon_e(\omega,\boldsymbol{q}) = -\frac{4\pi e^2}{\epsilon_\infty q^2}\Pi(\omega,\boldsymbol{q})$$

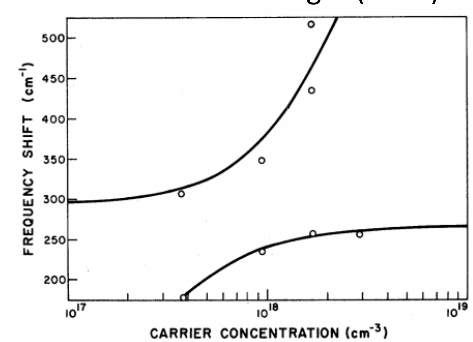


For $\boldsymbol{q} \to \mathbf{0}$ we have $\boldsymbol{\epsilon_e(\omega,q) \approx -\omega_p^2/\omega^2}$ and therefore:

1. For $\Omega_p \gg \omega_p$: same as undoped limit
2. For $\Omega_p \ll \omega_p$: $\omega_L \to \omega_T$ & transition is non-polar case

In STO $\boldsymbol{\Omega_p} \sim$ 97 meV – fits to option 1....

Moordian & Wright (1966)



Theory for FE QCP – the el-ph coupling

Fröhlich coupling -

$$L_{el-ph}^{LO} = -\sqrt{\frac{m}{n}}\frac{2\Omega_p\omega_p}{q^2}(\vec{q}\cdot\vec{u})c_{k+q}^{+}c_k$$

(only longitudinal mode)

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Spin-orbit coupling - $$L_{el-ph}^{SOC} = \alpha\left[c_{k+\frac{q}{2}}^{+}\left(\vec{k}\times\vec{\sigma}\right)c_{k-\frac{q}{2}}\right]\cdot\vec{u}_q$$

Kozii & Fu 2015 ; Gastiasoro et. al. 2020

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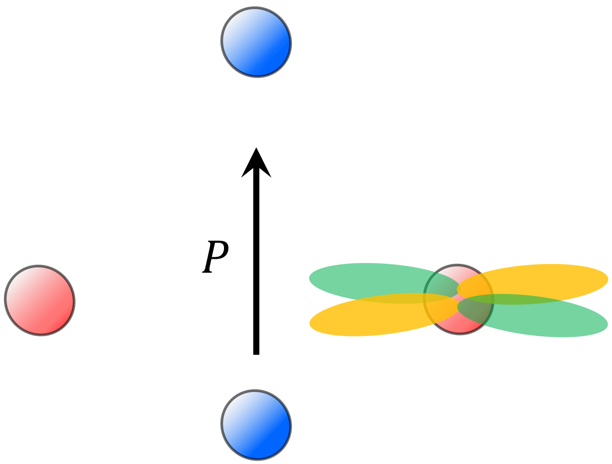

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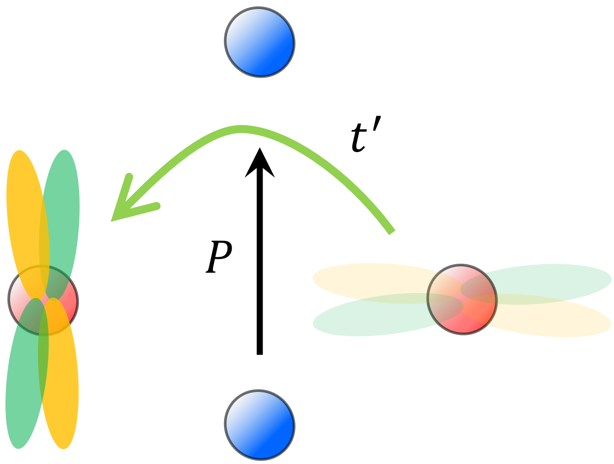

$$t' = \int d^3 r\, \phi_{xy}^{*}(\vec{r})\, P\, z\, \phi_{yz}(\vec{r}-\hat{x})$$

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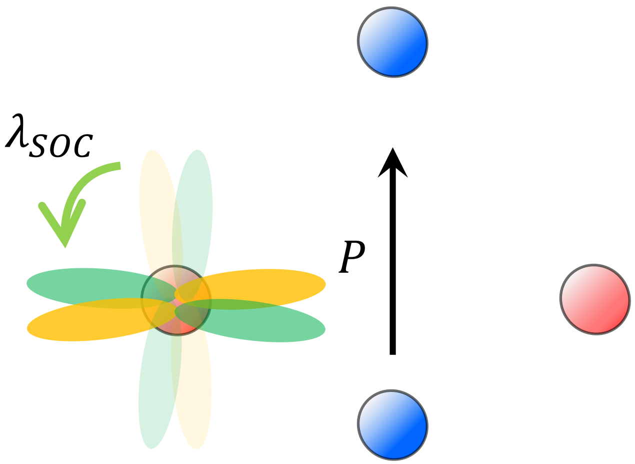


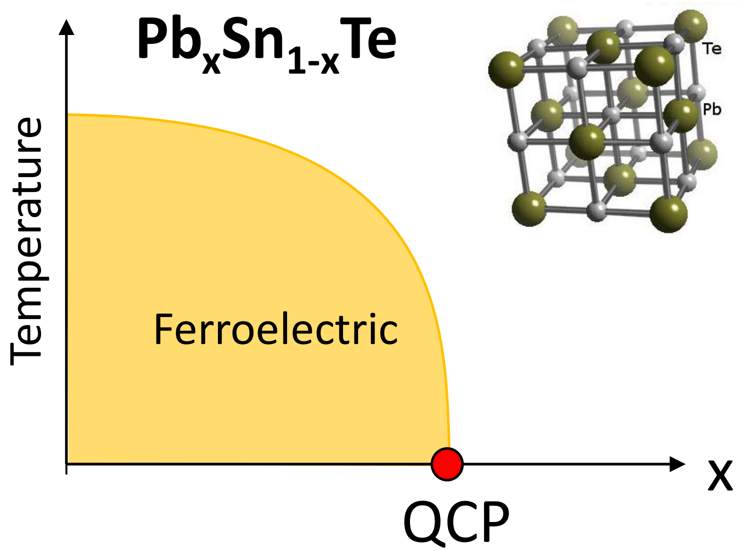
$$t' = \int d^3 r\, \phi_{xy}^{*}(\vec{r})\, P\, z\, \phi_{yz}(\vec{r}+\hat{x})$$

$$\alpha = \frac{t'\lambda_{SOC}}{\Delta}$$

*** Becomes small at $k_F \to 0$**

To understand such a theory we started from a Dirac point coupled to a FE QCP

PRX 9 031046 (2019)



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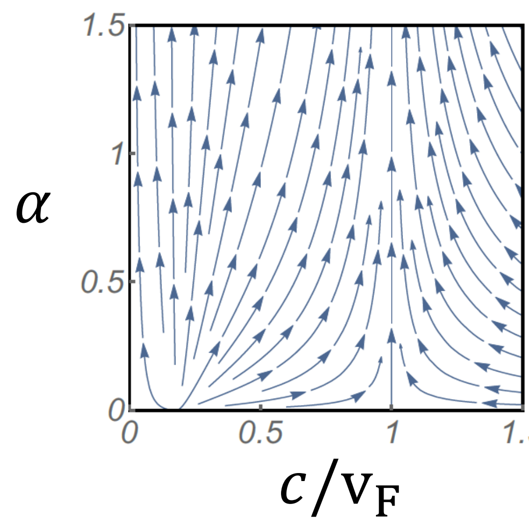
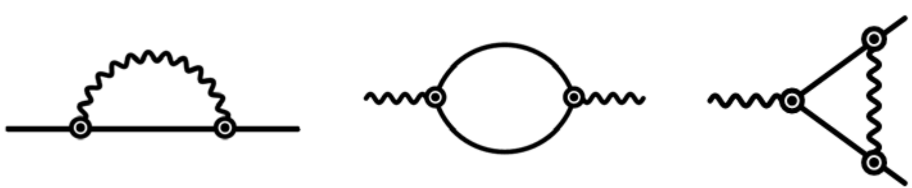
= ?

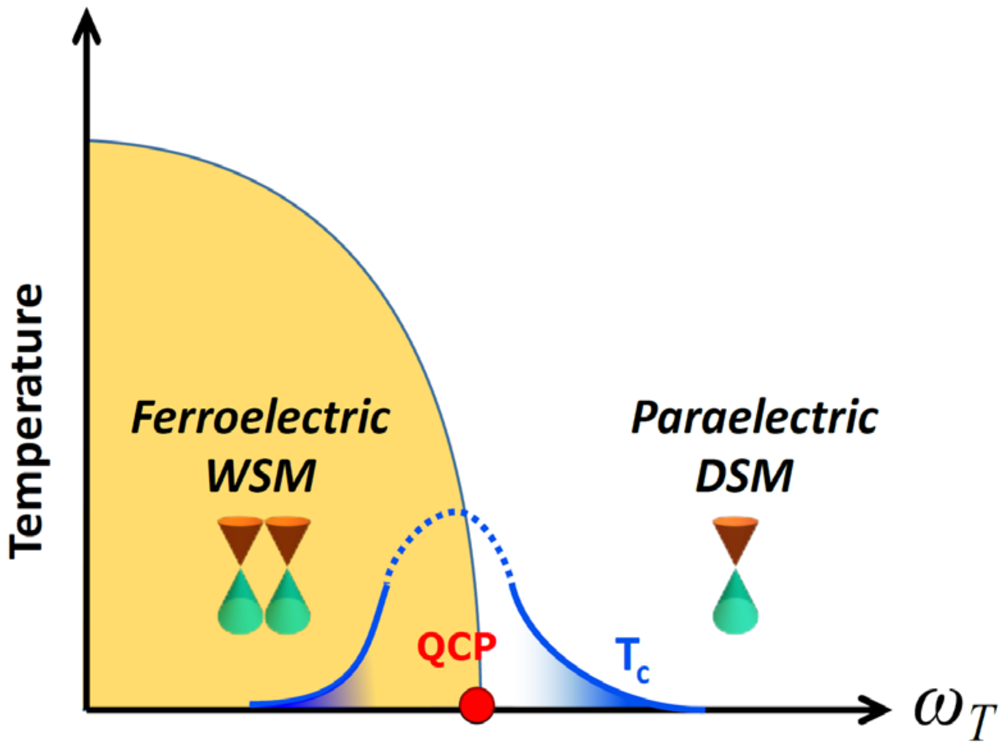
$$L_{el-ph}^{SOC} = \alpha\, \psi^{+}_{k+\frac{q}{2}}\, \vec{\gamma}\, \psi_{k-\frac{q}{2}} \cdot \vec{u}$$





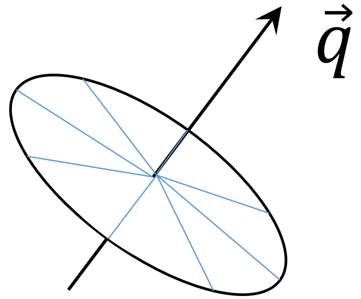
Removes Coulomb repulsion

Generates long-ranged interaction (gapless mode)



In summary, to understand the importance of FE fluctuations we need…

- **At q→0 Coulomb forces make soft mode transverse**

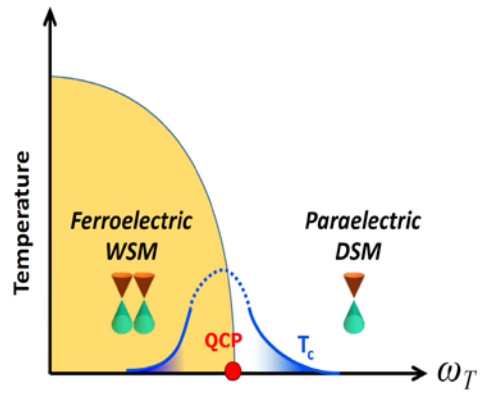


- **There is a direct coupling to through SOC**

$$L^{SOC}_{el-ph} = \alpha \left[c^{+}_{k+\frac{q}{2}} \left(\vec{k} \times \vec{\sigma} \right) c_{k-\frac{q}{2}} \right] \cdot \vec{u}_q$$

- **In Dirac systems in the relativistic limit this causes low-density SC.**

  (In STO one needs to check …)



- **Missing factors: disorder, phonon-phonon interaction, strong coupling effects …**

GLF theory to STO

Gurevich, Larkin and Firsov 1961, Takada 1980, JR & Lee 2016, Klimin et. al. 2018, Wolfle & Balatsky 2018, Rowely et. al. 2018 …

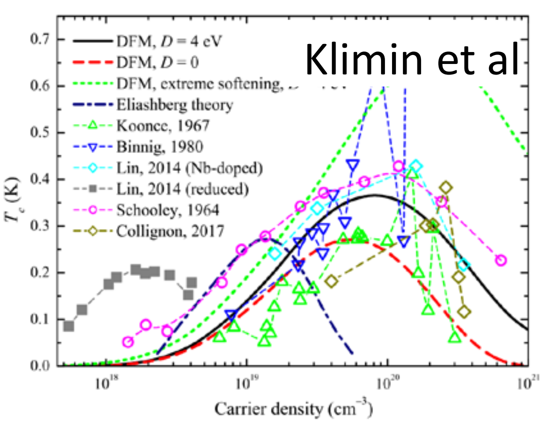
$$V_C(i\omega, \boldsymbol{q}) = \frac{4\pi e^2}{\epsilon_\infty [\epsilon_e(i\omega, \boldsymbol{q}) + \epsilon_c(i\omega, \boldsymbol{q})]\, q^2}$$

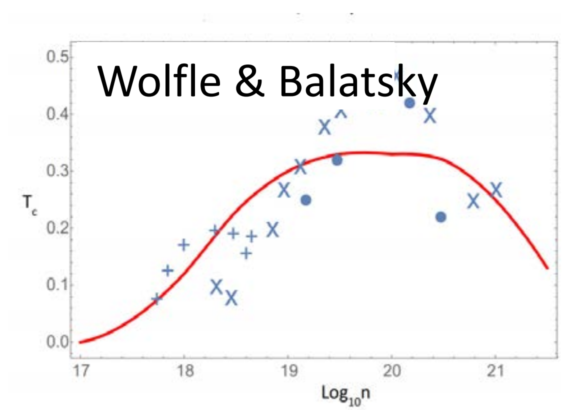
$$\epsilon_e(i\omega, \boldsymbol{q}) = -\frac{4\pi e^2}{\epsilon_\infty q^2} \Pi(i\omega, \boldsymbol{q})$$

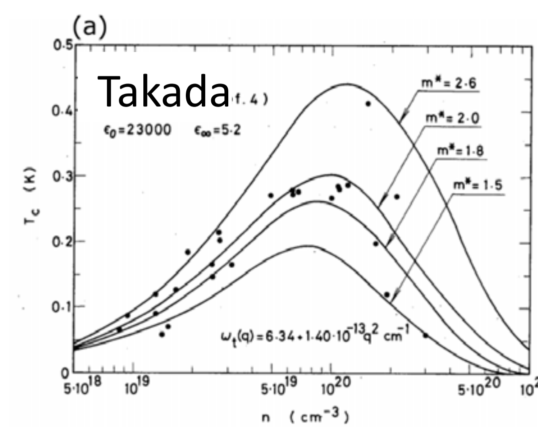
$$\epsilon_c(i\omega, \boldsymbol{q}) = \prod_{j=1}^{3} \frac{\omega_{Lj}^2(\boldsymbol{q}) - \omega^2}{\omega_{Tj}^2(\boldsymbol{q}) - \omega^2}$$

The s-wave pairing vertex

$$\Gamma(i\omega_n, k, p) = \frac{\nu}{4\pi} \oint d\Omega_{\boldsymbol{p}} V_C(i\omega_n, \boldsymbol{k} - \boldsymbol{p})$$







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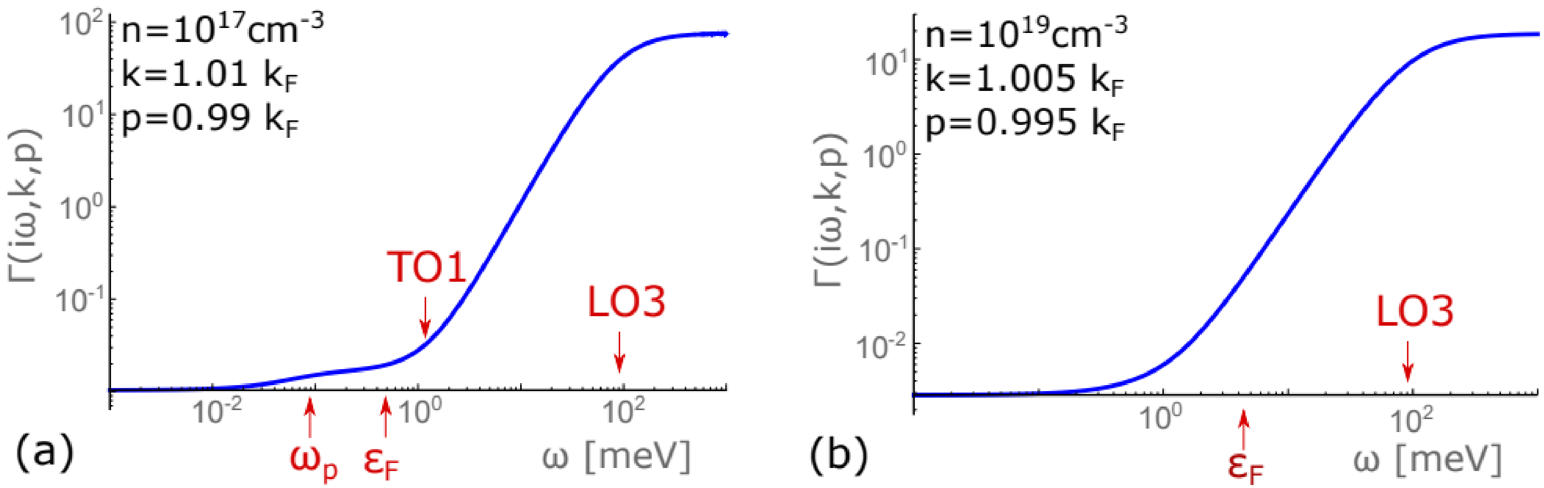


Gastiasoro, JR & Fernandes, 2020

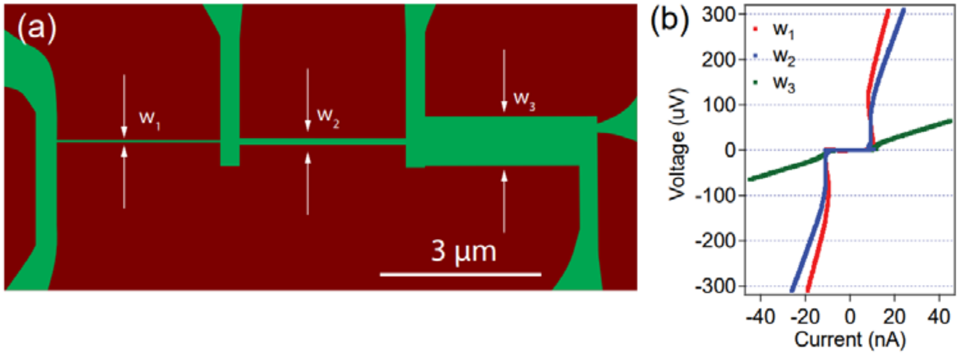
- **Works surprisingly well, but has issues of validity**
- **Predictive power, phenomenological consequences?**

Lower dimensional / filamental SC

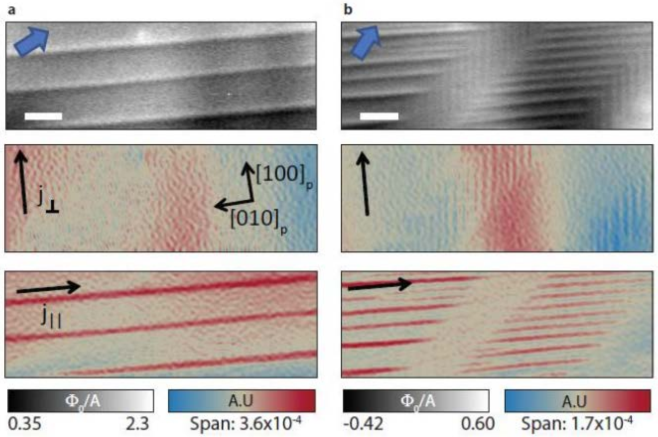
Lower dimensional states have larger density of states

$$N_{2D}(0) \sim \frac{m}{2\pi}$$

Jeremy Levy



Pai et. al. arxiv (2017)



Kalisky et. al. Nat. Mat. (2014)